

1. Crosswind Problem (suggested by Prof. Dunlap)

a. If you are driving at speed v and have 90° crosswind at speed u , how does the air resistance in the direction of the car's motion change, compared to the case with no crosswind? Assume high Reynold's number. (The engine doesn't care about the component of air resistance pushing the car sideways, only the component pushing it backward.)

b. What would be the answer in low Reynold's number limit?

c. In part a, for what angle of crosswind would you have no reduction in your gas mileage?

2. Projectile in linear medium. The equations of motion for a projectile in a linear medium are

$$m\ddot{x} = -bm\dot{x}$$

$$m\ddot{y} = -bm\dot{y} - mg$$

a) What is the terminal velocity for $v_{x0} = 0$?

b) Solve both equations for $x(t)$, $y(t)$ for initial v_{x0} , v_{y0} ; then eliminate t by solving the x equation for t and substituting into the y equation. Your (complicated) equation has x and y in it, but no t . It may clean up your algebra to use v_{ter} , and call $\tau = m/b$.

c) The range of the projectile is x when $y=0$. Replace x with R in your equation and set $y=0$. You now have a transcendental equation for R . To make some progress, assume low resistance (small k) and Taylor expand the logarithm. You should get a polynomial in $R = 0$.

d) Divide out R ($R = 0$ is a trivial solution); put R on one side and powers of R on the other. Now for the "trick"... put the solution for R when $k=0$ into the higher power(s) of R in your equation. Then you can express the range as a perturbation of the vacuum range, i.e. $R = R_{\text{vac}}(1-\epsilon)$ and find an expression for ϵ .

In this problem, only keep enough Taylor terms to get an answer different from the case of $b=0$. You don't need to keep lots of terms.

3. A tug-of-war is held between two teams, each consisting of 5 men. Each man has a mass of 100 kg. The men pull with force $F = 50 \text{ N exp}(-t/\tau)$, where the tiring time for team A is 20 seconds and team B is 10 seconds. Find the motion of the center of the rope.

What is the final velocity? What assumptions are unreasonable?

4. Consider a tetherball as it wraps around a pole. (Ignore gravity.)

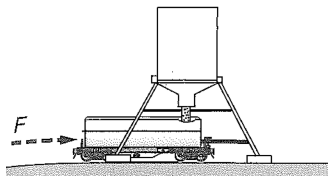
a.) Show that it is not possible for the ball to conserve both angular momentum *and* kinetic energy as the string wraps up.

b.) Is there a torque on the ball, if you take the center of the pole as the origin? (Hint: if the ball is

wrapping up closer to the pole, the pole must have a non-zero radius. You may take this radius to be small, *but not zero*.)

c.) At any given instant, the tension in the rope is the centripetal force. Use this, and $\tau = dL/dt$ to find a relationship between the distance the ball is from the pole and its angular velocity ω . Hint: you'll need to find a relationship between r (the distance the ball is from the pole) and θ , the angle it has moved through.

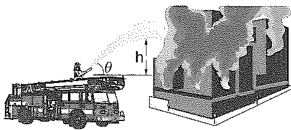
d.) Find $r(t)$. Does your result conserve energy or angular momentum?



4.11 Freight car and hopper*

An empty freight car of mass M starts from rest under an applied force F . At the same time, sand begins to run into the car at steady rate b from a hopper at rest along the track.

Find the speed when a mass of sand m has been transferred.



4.21 Force on a fire truck

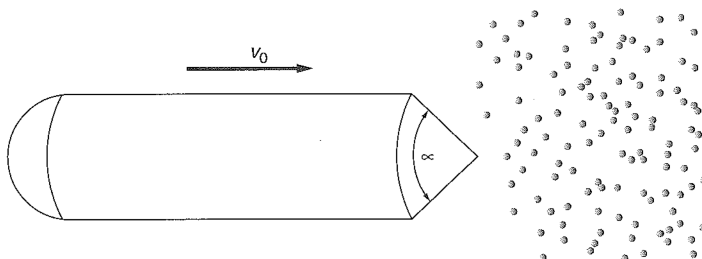
A fire truck pumps a stream of water on a burning building at a rate K kg/s. The stream leaves the truck at angle θ with respect to the horizontal and strikes the building horizontally at height h above the nozzle, as shown. What is the magnitude and direction of the force on the truck due to the ejection of the water stream?

4.26 Rocket in interstellar cloud

A cylindrical rocket of diameter $2R$ and mass M is coasting through empty space with speed v_0 when it encounters an interstellar cloud. The number density of particles in the cloud is N particles/ m^3 . Each particle has mass $m \ll M$, and they are initially at rest.

(a) Assume that each cloud particle bounces off the rocket elastically, and that the collisions are so frequent they can be treated as continuous. Prove that the retarding force has the form bv^2 , and determine b . Assume that the front cone of the rocket subtends angle $\alpha = \pi/2$, as shown.

(b) Find the speed of the rocket in the cloud.



• *Falling Rope.* A massive rope, length L , sits on a frictionless table. A small bit of rope, length x_0 , hangs through a hole in the table. Initially the rope is at rest and stretched out. Find the position of the end of the rope as a function of time.

In the situation described, as more rope falls through the hole, the entire rope moves. What equation would describe the dynamics if the rope were coiled so that only the rope falling through the hole moves? Would this rope fall faster or slower? Can you see that from your equations?